

Principles of Hydrology

Unit Hydrograph

Runoff hydrograph usually consists of a fairly regular lower portion that changes slowly throughout the year and a rapidly fluctuating component that represents the immediate response to rainfall.

The lower, slowly changing portion of runoff is termed base flow. The rapidly fluctuating component is called direct runoff. This distinction is made because the unit hydrograph is essentially a tool for determining the direct runoff response to rainfall.

Hydrograph components include rising limb, recession limb, peak, direct runoff, and baseflow.

1. The unit hydrograph is the direct runoff hydrograph produced by a storm of given duration such that the total volume of excess rainfall is 1 mm. The total volume of direct runoff is also 1 mm.
2. The ordinates of UH indicate the direct runoff flow produced by the watershed for every millimeter of excess rainfall; therefore, the units are $\text{m}^3/\text{sec}/\text{mm}$.
3. A volume of 1 mm is the amount of water in a 1-mm layer uniformly distributed over the entire watershed area. This volume is equal to the area under the UH.
4. Storms of different durations produce different UHs even if the excess rainfall volume is always 1 mm.
5. Longer storms will likely produce smaller peaks and longer duration in the UH.
6. The duration associated with the UH that of originating storm and not the base duration of the UH.

Assumptions

Unit hydrograph theory assumes that watersheds behave as linear systems. The following are the fundamental assumptions of UH theory.

1. The duration of direct runoff is always the same for uniform-intensity storms of the same duration, regardless of the intensity. This means that the time base of the hydrograph does not change and that the intensity only affects the discharge.
2. The direct runoff volumes produced by two different excess rainfall distributions are in the same proportion as the excess rainfall volume. This means that the ordinates of the UH are directly proportional to the storm intensity. If storm A produces a given hydrograph and Storm B is equal to storm A multiplied by a factor, then the hydrograph produced by storm B will be equal to the hydrograph produced by storm A multiplied by the same factor.

3. The time distribution of the direct runoff is independent of concurrent runoff from antecedent storm events. This implies that direct runoff responses can be superposed. If storm C is the result of adding storms A and B, the hydrograph produced by storm c will be equal to the sum of the hydrographs produced by storm A and B.
4. Hydrologic systems are usually nonlinear due to factor such as storm origin and patterns and stream channel hydraulic properties. In other words, if the peak flow produced by a storm of a certain intensity is known, the peak corresponding to another storm (of the same duration) with twice the intensity is not necessarily equal to twice the original peak.
5. Despite this nonlinear behavior, the unit hydrograph concept is commonly used because, although it assumes linearity, it is a convenient tool to calculate hydrographs and it gives results within acceptable levels of accuracy.
6. The alternative to UH theory is kinematic wave theory and distributed hydrologic models.

Unit Hydrograph Derivation

The discrete convolution equation allows the calculation of direct runoff for a given excess

$$Q_n = \sum_{m=1}^{n \leq M} P_m U_{n-m+1}$$

rainfall P_m

where n is the number of runoff steps, and m is the number of excess rainfall steps.

Example,

Synthetic Unit Hydrograph

1. Snyder=s Synthetic Unit Hydrograph

$$t_p = C_t (L L_c)^{0.3}$$

1. The basin lag is

where C_t is a coefficient ranging from 1.8 to 2.2, L is the length of the basin outlet to the basin divide, L_c is the length along the main stream to a point nearest the basin centroid.

2. The peak discharge rate is

$$Q_p = \frac{640 C_p A}{t^p}$$

where 640 will be 2.75 for metric system, C_p is a storage coefficient ranging from 0.4 to 0.8 where larger values of C_p are associated with smaller values of C_t , A is the drainage area.

$$T_b = 3 + t_p / 8$$

3. The base time is

For a small watershed, the base time is determined by multiplying t_p by a value ranging from 3 to 5.

$$D = t^p / 5.5$$

4. The duration for above equations is

$$t_{p'} = t_p + 0.25 (D' - D)$$

For other rainfall excess duration, the adjusted basin lag is

$$W_{50} = 770 (Q_p / A)^{-1.08}; \quad W_{75} = 440 (Q_p / A)^{-1.08}$$

5. The width equations for 50% and 75% of Q_p are

where 770 and 440 should be replaced with 2.14 and 1.22 when the metric unit system is used.

Example,

2. SCS method

The hydrograph is represented as a simple triangle with a rainfall duration D , time of rise T_R , time of fall

$$Vol = \frac{Q_p T_R}{2} + \frac{Q_p B}{2} \quad \vee \quad Q_p = \frac{2 Vol}{T_R + B}$$

B , and peak flow. The volume of direct runoff is

From the analysis of historical streamflow data, $B = 1.67 T_R$. So the peak discharge can be expressed

$$Q_p = \frac{0.75 Vol}{T_R}$$

as

$$Q(t) = \frac{(P(t) - I_a)^2}{(P(t) - I_a + S)}$$

The total runoff can be calculated by SCS curve number method.

$$S = \frac{1000}{CN} - 10$$

Where I_a is usually equal to $0.8 S$. The potential maximum retention S is defined as where CN is the curve number.

Example,

3. Application of unit hydrographs

1. Design storm hydrographs for selected recurrence interval storms (e.g., 50 yr) can be developed through convolution adding and lagging procedures.
2. Effects of land use-land cover changes, channel modifications, storage additions, and other variables can be evaluated to determine changes in the unit hydrograph.
3. Effects of the spatial variation in precipitation can be evaluated.
4. Hydrographs of watersheds consisting of several subbasins can be produced.

Example,

4. Other conceptual models

1. Instantaneous unit hydrographs
2. Linear models

4. Kinematic wave overland flow

Overland flow is formulated as a kinematic wave with a flow-direction algorithm to account for the overland-flow delay and storage in each grid cell. Mathematically, overland flow is described as

$$\frac{\partial d}{\partial t} + \frac{\partial q}{\partial x} = i_e, \quad q = a d^m$$

(Bedient and Huber, 1988):

where $d = d_{(x,t)}$ is depth of overland flow; $q = q_{(x,t)}$ is rate of overland flow; i_e is excess rainfall rate; a is conveyance; and m is surface roughness. Each grid cell and time step is further discretized into a large number of smaller segments to resolve kinematic-wave flow within a cell. On each grid cell, Equation 2

$$d_{(x,t)} = q\Delta t + d_{(x,t-1)} - am \left[\frac{\Delta t}{\Delta x} \right]^*$$

can be written explicitly in finite-difference form as

where Δt is a local time increment in THM residing in a global time step in HMS and Δx is the local distance increment within a single grid. On each grid cell, Equation A2 is solved numerically for the 1-D

$$\left[\frac{d_{(x,t-1)} + d_{(x-1,t-1)}}{2} \right]^{m-1} * [d_{(x,t-1)} - d_{(x-1,t-1)}]$$

flow. After depths of overland flow for segments within a cell are simulated, the water balance among inflow, outflow, and storage within each grid cell is determined. This finite difference solution provides hydrographs for overland flow on each grid cell. These hydrographs are added to produce collector hydrographs and, eventually, are transformed to channel or stream hydrographs. A conservative numerical scheme was implemented in THM to maintain the numerical stability in the case of $(c\Delta t/\Delta x) > 1$ (Bedient and Huber, 1988).

Clark Unit Hydrograph

Description of Method

The duration of a synthetic unit hydrograph generally is dependent upon the parameters used in the equations specific to the method. This differs from a derived unit hydrograph, as unit hydrographs derived directly from gaged data have a duration equal to the duration of excess precipitation from which they were derived. In this respect Clark's (1943) unit hydrograph is slightly different from other synthetic unit hydrograph methods in that it has no duration; it is an instantaneous unit hydrograph.

Clark's unit hydrograph theory maintains the fundamental properties of a unit hydrograph in that the sequence of runoff is the result of one inch of uniformly generated excess precipitation. However, the duration of the excess precipitation is considered to be infinitesimally small. Thus the Clark unit hydrograph generally is referred to as the Clark "Instantaneous Unit Hydrograph," or IUH. This excess precipitation is applied uniformly over a watershed which is broken into time-area increments.

This method is unique in its derivation, which draws from the parameters and theories of Muskingum hydrograph routing, and its application. The method was developed for gaged sites only, although later work has produced some suggestions for transfer of Clark parameters to ungaged sites.

Clark (1943) pointed out several advantages to his method, including the following:

1. The procedure is highly objective, using mathematically defined parameters based on observed hydrographs (except in the derivation of the time-area curve, which does require individual judgment). Thus, the procedure is repeatable by two individuals using the same data set.
2. The method does not require knowledge of spatial runoff distribution.
3. The ability to account for shape of drainage area and the capacity to produce large peak flows from concentrated runoff are included, subject to the accuracy of the developed time-area relationship.

4. The unit hydrograph is the result of an instantaneous rainfall with the time of concentration defined as the time between the end of rainfall and a mathematically defined point on the hydrograph's falling limb. Therefore, personal judgments on effective rainfall duration and time of concentration, which inversely affects peak flow, are eliminated.

Assumptions

In addition to the linearity assumptions common to all unit hydrograph methods, the Clark unit hydrograph method assumes that the runoff volume generated from each time-area increment is proportional to the area size of that increment.

Clark (1943) noted the following theoretical and practical limitations to his method:

1. Hydrographs are likely to underpredict on the recession side between the point of maximum recession rate and the point where subsurface flow begins to dominate.
2. The method may not be applicable to very large drainage areas. The application may cause too slow a rise and too rapid a recession, due to the use of the same storage factor for points which are both near to and far from the outlet. While Clark does not provide a recommendation on the maximum size of the drainage basin, he does indicate that the basin may be subdivided to overcome this drawback.
3. Finally, Clark admits that while the method appears to account for drainage basin shape and the capacity to produce high peak flows, it is possible that these influences are exaggerated by the method.

Data Requirements

Data required to develop the Clark unit hydrograph at a site include the discharge hydrograph and precipitation sequence for one or more large storm events, and a drainage map adequate to develop an estimate of travel time from various portions of the basin.

Development of Equations

A. Relation to Muskingum Routing Parameters

The Clark IUH is based on the premise that the unit hydrograph can be constructed from one inch of excess precipitation, which is then translated and routed through a reservoir to account for the storage effects of the basin. Therefore, Clark's original paper (1943) actually discusses much more than the procedure for deriving unit hydrographs. Clark also discusses valley storage and flood routing at great length, because of the dependence of his theory upon previous work with these concepts.

In his work, Clark used the Muskingum routing technique to investigate hydrographs for a number of gaged basins. Clark demonstrated that a hydrograph or flood wave, routed 5 times at 2-hour intervals with a Muskingum weighting coefficient of $x = 0.5$, could be approximated by the same original hydrograph or flood wave being translated 9 hours and then routed for a short time interval through a reservoir with a Muskingum weighting coefficient of $x = 0.0$. This is one of the basic concepts of the Clark Instantaneous unit hydrograph.

Besides the "x" weighting coefficient, the Muskingum routing technique requires a second coefficient, "K." The "K" parameter has units of time and is often presented as a reach travel time. More correctly, Clark referred to this parameter as a storage parameter, thus indicating the storage in terms of units of time (i.e., $K = 30$ hours of storage). References to the Clark unit hydrograph often denote the Muskingum K value, as used by Clark, as "R." As Muskingum "K," it represents storage effects in a routing reach; as Clark "R," it represents the same effects for a basin.

B. Development of Time-Area Histogram and Translation Hydrograph

The basin of interest is subdivided into time-area increments by estimating the time of travel to various locations within the drainage network and constructing isochrones (lines of equal travel time to the outlet). The area representing each increment of time is measured, and the total volume represented by one inch of runoff from that increment is calculated. Dividing by the time interval between the isochrones yields a volumetric flow rate, which is plotted against the travel time represented by the isochrones. In this way the "translation hydrograph," or sequence of flows resulting from the instantaneous generation of one inch of runoff from all time-area increments, is created.

C. Development of K coefficient

The difference between the time-area translation hydrograph and the basin outflow hydrograph is caused by basin storage and the resulting attenuation of the hydrograph. The Clark IUH relies on the Muskingum storage coefficient, K (now termed R in most references to the Clark method), to represent the attenuation imposed by basin storage properties. The following discussion retains the Muskingum “ K ” terminology, as Clark himself presented it in his original use of Muskingum routing to develop his theory.

Clark did not consider ungaged basins in his original work. Therefore, his method of estimating K relies upon gage information. He began by considering the three fundamental equations used in the Muskingum routing method:

$$dS = (I - O)dt \quad (1)$$

$$Q = xI + (1 - x)O \quad (2)$$

$$S = KQ \quad (3)$$

where

I = rate of inflow;

O = rate of outflow;

S = storage in reach or reservoir;

Q = weighted flow in the reach, in cfs;

x = Muskingum weighting coefficient; and

K = Muskingum storage coefficient

From these equations, it may be shown that:

$$K = \frac{dS}{dQ} = \frac{(I - O)dt}{xdI + (1 - x)dO} \quad (4)$$

and

$$O = I - Kx \frac{dI}{dt} - K(1 - x) \frac{dO}{dt} \quad (5)$$

Solving above equation via a finite difference formulation yields the familiar Muskingum formulation:

$$\frac{O_1 + O_2}{2} = \frac{I_1 + I_2}{2} - Kx \frac{I_2 - I_1}{\Delta t} - K(1-x) \frac{O_2 - O_1}{\Delta t} \quad (6)$$

which may be reduce to:

$$O_2 = C_0 I_2 + C_1 I_1 + C_2 O_1 \quad (7)$$

where:

$$C_0 = -\frac{Kx - 0.5\Delta t}{K - Kx + 0.5\Delta t} \quad (8a)$$

$$C_1 = -\frac{Kx + 0.5\Delta t}{K - Kx + 0.5\Delta t} \quad (8b)$$

$$C_2 = \frac{K - Kx - 0.5\Delta t}{K - Kx + 0.5\Delta t} \quad (8c)$$

Algebraically c_0, c_1, c_2 sum to 1.0. It is interesting to note that when the inflow has ceased ($I = 0.0$), Eq. 5 reduces to:

$$O = -K \frac{dO}{dt} (1-x) \quad (9)$$

and solving for K:

$$K = -\frac{O}{\frac{dO}{dt} (1-x)} \quad (10)$$

Clark felt that the ratio of $(dO/dt)IO$ should decrease until a minimum value is reached, and then remain constant. This probably would be true for a hydrograph that was the result of surface runoff only having no contribution from groundwater. Clark considered the shape of observed flood hydrographs and

noted that they do not actually behave this way. Instead, he found that the ratio would reach a minimum and then increase again. It is this minimum value of K that should be determined from an observed hydrograph and used in the routing of the translation histogram. Clark also uses this minimum value of K to define his time of concentration.

D. Time of Concentration (T_c)

Clark defines the time of concentration, T_c , as the time between the cessation of runoff-producing precipitation and the time at which this minimum value of K occurs. This time of concentration is used to produce the translation histogram. The time of concentration is then the maximum value used in the time-area relationship and the time-area curve is broken into smaller time intervals representing percentages of the time of concentration. Clark notes that this time-area curve delineation is indeed a trial-and-error procedure in many cases. However, he does not discuss the sensitivity of the time-area relationship further.

Clark's method relies on rainfall and flow data at the site of interest. He does not propose any other way (such as regressions or hydraulic estimates) to calculate the time of concentration. This limits the use of the method to gaged sites, although some later work investigated the transfer of Clark parameters from a gaged to an ungaged site, as discussed below.

E. Development of Hydrograph

Having determined the storage coefficient, K , the final routing of the translation histogram may now take place. The timestep used in the Muskingum routing is the time increment chosen for the time-area relationship. For reservoir type routing, Muskingum $x = 0.0$ and thus the coefficients become:

$$C_0 = C_1 = \frac{\Delta t}{2K + \Delta t} \quad (11)$$

$$C_2 = \frac{2K - \Delta t}{2K + \Delta t} \quad (12)$$

The IUH ordinates at time steps equal to the time-area increment then are determined by Eq. 7, where

I = the ordinates of the translation hydrograph and O = the ordinates of the IUH.

Use and Estimation of Parameters

The theoretical development above considers an instantaneous unit hydrograph (IUH) where the input rainfall has zero duration. In practical applications, it is usually necessary to alter the IUH into a unit hydrograph with a specific duration. This can be accomplished by lagging the IUH by the desired duration and averaging the ordinates. Thus for a two-hour unit hydrograph, lag or separate two IUH's by two hours and average the values of the ordinates at each time interval.

A. Derivation of Unit Hydrograph at a Gaged Site

The following steps summarize the application of the Clark method at a gaged site.

1. From a record of flood discharge, determine the time at which the rate of discharge decrease is greatest (i.e., $O/dO/dt$ is a minimum in relation to the prevailing discharge).
2. The time elapsed between the end of runoff-producing rainfall and the time so determined is T_c , or the base length (time) of a time-area concentration curve. The shape of the time-area curve is derived by dividing the watershed into increments of equal travel time along its water courses. Approximately 5 to 10 subdivisions are required to represent the time-area curve of the watershed. The more subdivisions, the more accurate the time-area relationship.
3. The storage coefficient K is determined by Eq. 10 at the time determined in step 1.
4. The time-area concentration curve is routed as if through a reservoir with Muskingum parameters K (determined in step 3) and $x = 0.0$. The routing is accomplished with equation 7, after deriving the coefficients C_0 , C_1 and C_2 from Eqs. 11 and 12.
5. The result, expressed in cubic feet per second per inch of runoff depth on the watershed at unit intervals of time, is the instantaneous unit hydrograph.
6. The instantaneous hydrograph is converted to a unit hydrograph for any desired duration by subdividing the instantaneous graph into periods of the desired unit length and averaging the ordinates over the preceding periods of time. For example, to derive a 6-hour unit

hydrograph, the rates of discharge at the end of 6, 12, and 18 hours, respectively, are the average ordinates to the instantaneous hydrograph in the 6-hour periods preceding the sixth, twelfth, and eighteenth hours.

B. Synthesis of Clark Parameters at an Ungaged Site

Clark used only gaged basins in his original work and did not provide guidelines for the estimation or determination of the K value and time-area relationships for ungaged basins. Clark specifically states that “the determination of any kind of unit hydrograph for a stream without records is a hazardous task”. In fact, Clark simply recommends that a stream gage should be installed at the point of interest. Although there are a number of methods and references for estimating the Muskingum coefficients for hydrologic routing in a channel system, the K value in the Clark IUH cannot be estimated in such a manner with any degree of confidence.

It is possible to transfer the K value from one basin to another nearby basin. This should be done with a regression analysis, as covered in Section 8-9.2.3 of the FERC *Guidelines*. (Note that the *Guidelines* use the more recent “R” designation for the parameter called K in Clark’s paper.) Parameters to be considered include (but are not limited to) drainage area, lengths, and slopes. The U.S. Army Corps of Engineers, Hydrologic Engineering Center, has noted that the ratio of $K/(T_c + K)$ tends to remain constant for a region. (HEC-1 User's Manual 1990a).

Most practitioners will compute Clark parameters from an observed storm with a computer program such as HEC-1. The only required input is the rainfall hyetograph and runoff sequence, with a user-estimated time-area curve being optional in HEC-1. HEC-1 also offers a standard time-area curve, It has been shown that these standard curves work reasonably well for a variety of conditions, provided that the basin is indeed of a similar nature to the ones used in the development of these curves. For information on synthetic time-area curves, the reader is referred to Section 3.3.2 of the HEC-1 Flood Hydrograph Package User’s Manual (1990).

Illustration

Figure 1, below, introduces an example basin which will be used to illustrate the development of the Clark unit hydrograph. Recall that the Clark IUH is based on 1 inch of excess precipitation being deposited instantly and uniformly over a basin. This basin is broken into subregions, which have been

delineated by locating isochrones (lines of equal travel time to the outlet) within the basin.