

HYDROGRAPH ROUTING

1. Introduction

A. Two Types of Hydrograph Routing

1. Storage or Reservoir Routing
2. Channel Routing

B. Reservoir Routing is used to determine the peak-flow attenuation that a hydrograph undergoes as it enters a reservoir or other type of storage pool.

C. Input data needed for storage routing include the inflow hydrograph and reservoir characteristics (storage and outlet facilities).

D. Channel Routing is used to analyze the effects of a channel on a hydrograph's peak flow and travel time.

E. Input data needed for channel routing include the inflow hydrograph and the channel characteristics

F. A typical highway application of hydrograph routing is the design of culverts. The design hydrograph is routed through the channel up to the stream crossing and then through the backwater that forms upstream of the culvert. The peak discharge, after the routing, may be used as the design flow for the culvert.

2. Channel Routing

A. When a hydrograph travels along a channel, its features change as a result of the channel's resistance and storage characteristics

B. In absence of other significant inflows along the reach, the peak flow is generally attenuated. This phenomenon is called diffusion and occurs in most natural channels

C. The time difference between the inflow and outflow peaks is a measure of the travel time in the reach

D. Given an inflow hydrograph and a channel, the objective of channel routing is to obtain the outflow hydrograph

B. The passing of a flood is essentially a wave traveling down the stream. Channel routing is essentially a wave analysis.

3. Fundamental Equations

G. Computational methods are based on Principles of Conservation of Mass (Continuity Equation) and Conservation of Linear Momentum Equations.

H. The Continuity equation expresses conservation of mass which states that the change in storage is equal to the inflow minus the outflow.

$$I - O = \frac{dS}{dt}$$

Where t is the time in seconds and S is the storage in cubic meters, which is the volume contained between two sections of the channel. I is the inflow in cubic meters per second consisting of the incoming flow and any other addition to the reach between the two sections (e.g., overland flow, tributaries, and outfalls). O is the outflow in cubic meters per second which includes the outgoing flow and any other water loss (e.g., infiltration through the channel bottom, diversions, and withdraws).

I. The momentum equation, also called the dynamic equation, is the same as Newton's second law which states that the change in momentum is equal to the sum of all the applied forces on the control volume.

$$\frac{d(mv)}{dt} = \Sigma F$$

Where m is the mass (kg) and v the velocity (m/s) of the water volume between the two channel sections. The sum of the forces F includes pressure, friction, and gravity terms.

J. Routing involves the solution of the continuity and momentum equations after making some assumptions to simplify the calculations

The original Saint Venant equations include the conservation of mass:

$$\frac{\partial(AV)}{\partial x} + \frac{\partial A}{\partial t} = 0$$

and the conservation of momentum:

$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + g \left(\frac{\partial h}{\partial x} + S_f \right) = 0$$

where

A = cross-sectional area of the channel, in square feet (ft²);

V = velocity, in feet per second (fps);

x = distance along channel, in feet (ft);

t = time, in seconds (s);

g = acceleration due to gravity, in feet per second per second (ft/s²);

h = water surface elevation, in feet;

S_f = friction slope, in ft/ft.

The equations are considered quasi-linear hyperbolic partial differential equations containing dependent variables (V and h) and independent variables (x and t). Depth of water and channel geometry combine to define channel cross-sectional area. Friction slope is typically defined by the Manning or Chezy equations and is dependent on velocity and depth of water.

Simplifications and approximations of the Saint Venant equations can be separated into four categories: empirical, linearized, hydrological, and hydraulic (Fread, 1985). Empirical methods are based on a large collection of observed data for a discrete reach of river and are only applicable for the reach. Linearized methods simplify the Saint Venant equations by neglecting nonlinear terms, the remaining terms; using these assumptions, an analytical solution can be obtained. Hydrologic methods utilize the conservation of mass equation and a relationship between storage and discharge. Hydraulic methods incorporate the conservation of mass equation and simplified forms of the conservation of momentum equation.

5. Empirical and Linear Models

This document focuses only briefly on empirical and linear methods, as their use is not as common today as the use of hydrologic and hydraulic methods.

Empirical models are derived from observed flow and stage data collected from a discrete section of a river. It is essential that the data include a range of flows from low to high to calculate an accurate empirical relationship or routing coefficients. The most accurate results are obtained from empirical methods when applied to slowly fluctuating rivers with negligible lateral inflows and backwater effects (Fread, 1985). The two most common types of empirical methods are Lag and Gage methods.

Lag methods assume that average inflows occur at a later time further downstream. The Successive Average and Progressive Average lag methods are the most common. The Successive Average method assumes that outflow is based on a specific number of averaged inflows within the reach. Tatum (Fread, 1985) found that the number of successively averaged inflows is approximately equal to two times the travel time divided by the length of the reach. Outflow is computed by:

$$O_{n+1} = C_1 I_1 + C_2 I_2 + \dots + C_{n+1} I_{n+1}$$

where n equals the number of successive averages within the reach. The routing coefficients C_1, C_2, \dots, C_{n+1} can be calculated by trial and error using observed inflow and outflow hydrograph data. The Progressive Average method also averages successive inflow values but lags them by the travel time of the flood wave. By varying the period of the inflow flood wave, the resulting calculated outflow hydrograph can be adjusted to match an observed outflow hydrograph.

The empirical relation that associates flow at a downstream section to an upstream section is referred to as a gage relation. Gage relationships typically involve both flow and stage values in which lateral inflows and backwater effects are inherent. Linsley et al. (1949) reported that the following two factors are critical to the success of the method: (1) lateral inflow between the upstream and downstream sections must be proportional to the flow at the upstream section, and (2) a fixed time relation must exist between the peak flow at the upstream section and the peak of the inflow in the interjacent area. They concluded that if these conditions are not violated, the method will produce accurate results. However, as actual conditions begin to diverge from these requirements, a more elaborate and intricate relation may become necessary.

Linearized models simplify the Saint Venant equations by neglecting or linearizing certain terms. This enables the equations to be integrated and solved analytically. Typical simplifying assumptions include (Miller and Cunge, 1975):

1. The velocity head term, $V(MV/Mx)$, is considered negligible.
2. Cross-sectional area is considered to be constant, typically rectangular.
3. Channel bed slope is assumed constant, often zero.
4. The friction slope term is linearized with respect to velocity and depth.
5. There is no lateral inflow.
6. The flood wave has a simple shape that can be expressed analytically.

Henderson (1966) relates through experimentation and experience that for steep slopes the $V(MV/Mx)$, MV/Mt , and Mh/Mx terms can be omitted due to their relative insignificant values. Henderson also found that for very flat slopes the $V(MV/Mx)$ and MV/Mt terms can be neglected but the MV/Mx term cannot, since it may be of the same magnitude as St . Although it cannot be neglected, the Mh/Mx term may be linearized (Miller and Cunge, 1975).

Linear methods simplify the Saint Venant equations so that analytical solutions may be obtained. However, the simplifying assumptions that are made impose strict limitations which may invalidate the solution, and as a consequence, make the linear methods impractical for use.

6. Hydrologic Methods

Hydrologic routing methods were developed as the need for simplified routing techniques became apparent. Hydrologic methods are based on the concept that inflow, outflow, and storage must adhere to the conservation of mass principle, described by the following equation:

$$I - O = \frac{\Delta S}{\Delta t}$$

- I = inflow to the reach, in cfs,
 O = outflow from the reach, in cfs,
 $\hat{I} S$ = change in storage within the reach, in cubic feet,
 $\hat{I} t$ = time increment, in seconds.

Hydrologic methods can effectively reproduce flood flows when a storage-discharge relation is calculated or routing coefficients are fitted to the storage-discharge relation. However, the computed relation is typically single valued whereas the dynamic properties of a flood wave create a looped relation between storage and discharge. This is apparent when considering that the transverse water surface slope and storage are greater during the rising stages of a flood wave than during the falling stages. Consequently, limitations to the methods exist; they include rivers with significant backwater effects, tributary inflows, and flat to mild channel slopes among others (Bedient and Huber, 1992; Fread, 1985). Figure 1 depicts a looped storage-discharge relation (called hysteresis)

6.1. Modified PULS

Description of Method

Modified Puls routing utilizes the simple concept that storage is a function of outflow. Correct computation of the outflow hydrograph rests on the assumption that storage depends primarily, if not solely, on outflow rate. For this reason, Modified Puls routing is typically used for reservoir routing where a unique storage-outflow relation is likely. Strelkoff (1980) stated that determination of this relationship is a key factor in the application of the Modified Puls method.

To perform the routing, a relationship between storage and outflow is calculated and plotted as a curve. The following form of the continuity equation is then solved for each time step.

$$\left(\frac{S_2}{\Delta t} + \frac{O_2}{2}\right) = \left(\frac{S_1}{\Delta t} + \frac{O_1}{2}\right) - O_1 + \left(\frac{I_1 + I_2}{2}\right)$$

Modified Puls routing proves valid for reservoirs when the effects of a flood wave (differences in storage due to rising and falling stages) are dampened, if not eliminated, by the reservoir. Modified Puls can be used for channel routing in a similar manner where each subsection of the reach is considered to behave like a cascading reservoir. Some error is inherent in this concept since storage in a river reach is not a function of outflow alone.

Assumptions

When using the Modified Puls method, it is assumed that a unique and single-valued stage-storage-outflow relationship exists for each reach, and that changing downstream conditions will not alter this relationship.

Limitations

The method is not recommended for (1) channels with gradients less than 3 feet/mile, (2) reaches with time varying downstream boundaries such as tidal influences; or (3) rapidly rising flood hydrographs such as dam breaks (HEC, 1990).

A report presented to the Army Corps of Engineers by Strelkoff (1980) suggests that great caution should be used when applying the Modified Puls method to mild or flat slopes. Strelkoff found that the storage-discharge relation was highly event dependent. Due to the flatness of the channel, he determined that a substantial depth gradient is required to propagate the flood flow. Therefore, during the beginning of flood inflow, a large percentage of flow fills storage areas (floodplains) until the depth gradient is attained. Small changes in outflow are observed while this is occurring. As the flood wave moves downstream, the outflow increases markedly but little storage is added. Thus, storage-discharge relationships calculated by steady flow profiles produce errors, which may be severe when out-of-bank flows occur over wide flood plains. Calibrated storage-discharge relationships also produce errors because they force a single-valued relation and because each event develops a different looped relation.

Data Requirements

The Modified Puls method requires either a known stage-storage-discharge relationship, or hydraulic geometry data adequate to calculate this relationship for each reach. An appropriate computation time step also must be selected, which requires an estimate of the travel time through the reach.

Development of Equations

The heart of the Modified Puls equation is found by considering the finite difference form of the continuity equation (5) which may be written as:

$$\frac{I_1 + I_2}{2} - \frac{O_1 + O_2}{2} = \frac{S_2 - S_1}{\Delta t}$$

After an algebraic transformation, The above equation is written:

$$I_1 + I_2 + \left(\frac{2S_1}{\Delta t} - O_1\right) = \frac{2S_2}{\Delta t} + O_2$$

In the equation above, the left side is known at a given time, while the right side is to be calculated. Basically, the solution to the Modified Puls method is accomplished by developing a graph (or table) of O versus $[2S/\Delta t + O]$. In order to do this, the relationship between outflow O and storage S must be known, assumed, or derived.

Use and Estimation of Parameters

The Modified Puls method is available in the HEC-1 Flood Hydrograph Package. Another common method of applying this method for river simulation is to use a combination of HEC-1 and the HEC-2 Water Surface Profile Program.

The HEC-1 program allows the user to enter the storage-outflow relationships directly, or they may be computed from eight-point cross-sectional data provided to the model. If cross section data are entered, the normal depth is calculated for each cross-section using Manning's equation. The danger in using this method is that downstream effects cannot be taken into account for each cross-section. Also, if this eight-point cross-section is not truly representative of the reach, then the stage-storage relationships cannot be developed accurately.

Steady flow profile computations performed at varying flows can be used to simulate the cascading reservoirs. Also, the relation for a reach can be derived from observed inflow and outflow data from that reach. When using the HEC-2 Water Surface Profile program in conjunction with HEC-1, HEC-2 is used to calculate water surface profiles for a series of flows. The outflow and storage for each reach is computed from the backwater computations. This table of information is then written to a file which may be retrieved by the HEC-1 program. The HEC-1 program uses the outflow and storage information developed by the HEC-2 program to develop the O -vs- $[2S/\hat{I} t + O]$ relationships.

Care should be taken when applying this method and selecting the time increment, delta t. The delta t time increment should be less or approximately equal to the travel time through the reach. Criteria have been developed regarding the number of reach subsections and the routing time step, delta t. Typically, reach length, average channel velocity, and time interval determine the number of reach subsections (Hydrologic Engineering Center, 1990a). Brunner (1992) states that the number of steps (reach subsections) affects the attenuation of the hydrograph and should be obtained by calibration. Maximum attenuation occurs when channel routing is performed in one step. Conversely, hydrograph attenuation decreases as the number of routing steps (reach subsections) increase. The routing time interval can be estimated as 20% of the time to rise of the inflow hydrograph (Hydrologic Engineering Center, 1990a).

6.2. The Muskingum Method

A. This method ignores the momentum equation and is based solely on the continuity equation. The method is applicable to diffusion waves

B. The peak is attenuated as a result of diffusion caused by storage effects. The storage is assumed to be given by

$$Q = xI + (1 - x)O$$

$$S = KQ$$

Where K is the travel time in seconds between the two channel sections, and x is a dimensionless factor between 0.0 and 0.5 that weights the influence of the inflow and outflow hydrograph to the storage within the reach

C. If $x = 0.5$, the storage depends equally on inflow and outflow. If $x = 0$, the storage depends only on the outflow, as in the case of a large body of water such as a reservoir

D. Substituting the storage equation into the continuity equation yields

$$O_2 = C_0 I_2 + C_1 I_1 + C_2 O_1$$

where:

$$C_0 = -\frac{Kx - 0.5\Delta t}{K - Kx + 0.5\Delta t}$$

$$C_1 = \frac{Kx + 0.5\Delta t}{K - Kx + 0.5\Delta t}$$

$$C_2 = \frac{K - Kx - 0.5\Delta t}{K - Kx + 0.5\Delta t}$$

and the sum of all coefficients is 1.

A. The most critical part of the calculation is to estimate suitable values of K and x. These values should be obtained by calibrating to available sets of measured inflow and outflow hydrograph data for the channel reach. If this is not possible a value x between 0.2. $<x < 0.5$ is recommended

B. Once K and x are known for a channel reach, the computational procedure to obtain the outflow hydrograph is as follows:

1. Discretize the inflow hydrograph in time increments of Δt
2. Calculate the three coefficients
3. Use the Muskingum equation to compute the outflow hydrograph at the end of the channel reach
4. Repeat step 3 until the end of the inflow hydrograph has been reached

6.3. Muskingum-Cunge Method (Diffusion Waves)

A. Most flood waves experience attenuation and are better approximated by diffusion waves rather than by kinematic waves

B. The momentum equation takes into account the slope of the water surface allowing nonuniform flow to take place in the channel. The governing equation can be written as

$$\frac{\partial Q}{\partial t} + c \frac{\partial Q}{\partial x} = \frac{q_0}{2S_0} \frac{\partial^2 Q}{\partial x^2}$$

Where q_0 is an average discharge per unit channel width (m^2/sec) and all other variables are as defined previously

C. A diffusion wave should satisfy the following inequality

$$T_r S_0 \left(\frac{g}{d_0} \right) \geq 15$$

Where t_r (sec) is the time-to-peak of the inflow hydrograph, S_0 the channel slope, d_0 the average flow depth in meters, and g the gravitational constant equal to 9.81 m/s^2 .

D. Computational method for diffusion waves is the Muskingum-Cunge method:

$$O_2 = C_0 I_2 + C_1 I_1 + C_2 O_1$$

where:

$$C_0 = \frac{-1 + C + D}{1 + C + D}$$

$$C_1 = -\frac{1 + C - D}{1 + C + D}$$

$$C_2 = 1 - C_0 - C_1$$

and

$$C = c \frac{\Delta t}{\Delta x}$$

$$D = \frac{q_0}{S_0 c \Delta X}$$

E. The form of the routing equation is the same as in the form of the Muskingum method; only the definition of the coefficients is different

F. The celerity c is based on the peak flow

G. In the Diffusion coefficient D , q_0 is a reference flow per unit width ($\text{m}^3/\text{s}/\text{m}$), for instance, the average flow divided by the average width. The flood wave is considered to be a perturbation with respect to this reference discharge

H. Optimal results are obtained if C is close to and not greater than unity, and if the sum of C and D is greater than 1. In addition t_r / \hat{t} should be greater than 5

J. The Muskingum method is a special case of the Muskingum Cunge method. Unlike the Muskingum method, the Muskingum-Cunge requires minimal streamflow data

K. The kinematic wave method is identical to the Muskingum-Cunge method when $D=0$

7. Hydraulic Methods

Hydraulic routing employs the full dynamic wave (St. Venant) equations. These are the continuity equation and the momentum equation, which takes the place of the storage-discharge relationship used in hydrologic routing. The equations describe flood wave propagation with respect to distance and time. Henderson (1966) rewrites the momentum equation as follows:

$$S_f = S_o - \left(\frac{\partial y}{\partial x}\right) - \left(\frac{V\partial V}{g\partial x}\right) - \left(\frac{1}{g} \frac{\partial V}{\partial t}\right)$$

where

S_f = friction slope (frictional forces), in ft/ft;

S_o = channel bed slope (gravity forces), in ft/ft;

Second term = pressure differential;

Third term = convective acceleration, in ft/sec²;

Last term = local acceleration, in ft/sec²

Henderson also writes the continuity equation as follows:

$$A \frac{\partial V}{\partial x} + VB \frac{\partial y}{\partial x} + B \frac{\partial y}{\partial t} = q$$

The description of each term:

$A(MV/Mx)$ = prism storage

$VB(My/Mx)$ = wedge storage

$B(My/Mx)$ = rate of rise

Q = lateral inflow

The full dynamic wave equations are considered to be the most accurate solution to unsteady, one-dimensional flow but are based on the following assumptions used to derive the equations (Henderson, 1966):

1. Velocity is constant and the water surface is horizontal across any channel section.
2. Flows are gradually varied with hydrostatic pressure prevailing such that vertical acceleration can be neglected.
3. No lateral circulation occurs.
4. Channel boundaries are considered fixed and therefore not susceptible to erosion or deposition.

5. Water density is uniform and flow resistance can be described by empirical formulas (Manning, Chezy)

Solution to the dynamic wave equations can be divided into two categories: approximations of the full dynamic wave equations, and the complete solution.

The three most common approximations or simplifications to the full dynamic equations are referred to as kinematic, diffusion, and quasi-steady models. They assume certain terms of the momentum equation can be neglected due to their relative orders of magnitude. The full momentum equation is

$$S_f = S_o - \left(\frac{\partial y}{\partial x}\right) - \left(\frac{V\partial V}{g\partial x}\right) - \left(\frac{1}{g} \frac{\partial V}{\partial t}\right)$$

Kinematic and diffusion models have found wide application and acceptance in the engineering community (Bedient and Huber, 1988). This acceptance can be attributed to their application to mild and steep slopes with slow rising flood waves (Ponce et al., 1978). Henderson (1966) supported this by computing values for each term in the momentum equation. He found that the last three terms of the momentum equation are two orders of magnitude less than the channel bed slope value and therefore are negligible for steep slopes.

7.1. Kinematic Waves

- A. The kinematic wave method assumes that the motion of the hydrograph along the channel is controlled mostly by gravity and friction forces. Therefore, uniform flow is assumed to take place. The momentum equation becomes a wave equation

$$\frac{\partial Q}{\partial t} + c \frac{\partial Q}{\partial x} = 0$$

Where Q is the discharge, t the time, x the distance along the channel, and c the celerity of the wave (speed).

- B. A kinematic wave travels downstream with speed c without experiencing any attenuation or change in shape. Therefore, diffusion is absent.

- C. Kinematic wave behavior can be assumed if

$$\frac{t_r S_0 V}{d_0} \geq 85$$

Where t_r is the time-to-peak of the inflow hydrograph, and the other terms are defined as before.

- D. Kinematic waves are likely to occur in steep channels and with long duration hydrographs
- E. The outflow equation can be discretized in time and space to yield

$$O_2 = C_0 I_2 + C_1 I_1 + C_2 O_1$$

where:

$$C_0 = \frac{C-1}{1+C}$$

$$C_1 = 1$$

$$C_2 = \frac{1-C}{1+C}$$

and

$$C = c \frac{\Delta t}{\Delta x}$$

F. From the Manning formula, the celerity speed, c , in a wide channel is

$$C = (5/3)V$$

G. If the channel cannot be considered wide, the celerity is given by $(dQ/dy)/T$ where T is the top width of the channel, y is the stage or depth, and Q is the discharge. Therefore, dQ/dy is the slope of the rating curve.

H. C is the Courant number. The distance and time steps Δx and Δt must be selected such that C is equal to unity. Otherwise, numerical errors may result

I. The routing equation has the same form as in the Muskingum method. The definition of the routing coefficients is different for each method.

J. The computational procedure is as follows:

1. Discretize the channel reach according to a chosen $\hat{\Delta} x$ and determine the initial flow conditions
2. For every spatial interval, use Manning's formula to estimate the flow velocity V and calculate c as $5/3V$
3. Set $C=1$ and calculate $\hat{\Delta} t$ from the Courant relationship for all spatial intervals

$$\Delta t = \frac{C\Delta x}{c}$$

select $\hat{\Delta} t$ as the minimum of all spatial intervals in each reach

4. Calculate the coefficients

5. Use the routing equation to compute the outflow hydrograph at the next time step for all spatial intervals
6. Repeat all the steps until reaching the last time step

7.2. The Modified Att-Kin Method

- A. The attenuation-kinematic (Att-Kin) method is a combination of the storage indication method (reservoir routing) and the kinematic wave methods
- B. The hydrograph is first routed through a reservoir assuming that the outflow is proportional to the storage. The resulting outflow hydrograph is then translated using kinematic wave principles
- C. The storage routing portion allows for attenuation and translation. Kinematic routing translates the outflow hydrograph but does not attenuate the peak.
- D. The time of occurrence of the peak coincides with the maximum storage in the channel reach.
- E. This method is used in the TR-20 and TR-55 methodologies of the SCS (Now the Natural Resources Conservation Service (NRCS))
- F. Steps in Storage Routing - STEP 1

1. The continuity equation is discretized as

$$I_1 - \frac{O_2 - O_1}{2} = \frac{S_2 - S_1}{\Delta t}$$

Where delta represents a time step, subscript 1 denotes values at the beginning of the time step and subscript 2 indicates values at the end of the time step

2. Substitution of $S = KO$ leads to

$$O_2 = C_m I_1 + (1 - C_m) O_1$$

Where

$$C_m = \frac{2\Delta t}{2K + \Delta t}$$

3. The coefficient K is the wave travel time in the reach and is defined as $K = L/c$
4. L is the channel reach length, c = my the celerity of the wave, v is the mean velocity, and m is related to the rating curve of the cross section

$$Q = xA^m$$

5. Logarithmic linear regression can be performed on discharge versus area data to determine x and m
6. In the absence of data, m can be assumed equal to 5/3, and

$$x = \frac{S_0^{1/2}}{nP^{2/3}}$$

Which stems from Manning's formula for a channel with S_0 , roughness coefficient n, and wetted perimeter P

G. Kinematic Wave Translation - STEP 2

1. After the storage routing is performed, the kinematic travel time is estimated from

$$\Delta t_p + \frac{Q_p^{(1/m-1)}}{x^{1/m}} L \left(\frac{(I_p / O_p)^{1/m} - 1}{(I_p / Q_p) - 1} \right)$$

Where I_p , is the peak inflow and O_p , the peak outflow

H. The computational procedure is as follows:

1. Determine x and m from the rating curve for representative cross section
2. Calculate K and C_m . Select delta t such that C_m less than unity (preferably $C_m \leq 0.67$)
3. Perform storage routing
4. Calculate delta t_p and compare with the time difference between inflow and outflow peaks $\hat{T} t_{ps}$
5. If $\hat{T} t_p \leq \hat{T} t_{ps}$ translate the outflow hydrograph by $\hat{T} t_p - \hat{T} t_{ps}$. Otherwise, the hydrograph stays the same

Table Summary of Features of Channel Routing Models

1. In practice, a computer model such as TR-20 or HEC-1 will be used to solve problems involving routing. TR-20 utilizes the Att-Kin method only for channel routing. The HEC-1 model offers several choices.
2. The best hydrologic routing procedure is the Muskingum-Cunge because it includes both the Muskingum and kinematic wave features.

7.3. Diffusion Wave Routing

Description of Method

The diffusion wave approximation includes the pressure differential term but still considers the inertial terms negligible; this constitutes an improvement over the kinematic wave approximation. The diffusion wave approximation is shown below.

$$S_f = S_o - \frac{\partial y}{\partial x}$$

The pressure differential term allows for diffusion (attenuation) of the flood wave and the inclusion of a downstream boundary condition which can account for backwater effects.

Assumptions

The diffusion wave method is based on the assumption that the inertial terms in the momentum equation are negligible. This is appropriate for most natural, slow-rising flood waves but may lead to problems for flash flood or dam break waves. The assumptions common to all hydraulic routing methods (one-dimensional flow, stable channel, and constant density) apply to this method.

Limitations

Since the inertial terms are not included in the approximation, the method is limited to slow to moderately rising flood waves (Fread 1982). According to Brunner (1992), most natural flood waves can be described with the diffusion form of the equations. However, Lettenmaier and Wood (in Maidment, 1992) recommend limiting the method to rivers without significant backwater effects and with slopes greater than 0.05.

Data Requirements

Diffusion wave routing is not supported by the HEC-1 model but is supported by some dynamic routing models such as NWS DAMBRK/FLDWAV. The data requirements are similar to those for full dynamic models (see below), including cross sections, slope, and hydraulic roughness of the channel and overbank.

Development of Equations

The diffusion wave method combines the continuity equation.

Use and Estimation of Parameters

Diffusion wave routing is accomplished by combining and solving equations. Manning's equation is used to estimate the friction slope S_f . Therefore, the channel must be described as a series of cross sections that adequately describe the hydraulic character of the entire flow path, including slope, geometry, and roughness.

7.4. Quasi-Steady Dynamic Wave Routing

Description of Method

The quasi-steady dynamic wave approximation method incorporates the convective acceleration term but not the local acceleration term, as indicated below:

$$S_f = S_o - \left(\frac{\partial y}{\partial x}\right) - \left(\frac{V\partial V}{g\partial x}\right) \quad (38)$$

In channel routing calculations, the convective acceleration term and local acceleration term are opposite in sign and thus tend to negate each other. If only one term is used, an error result which is greater in magnitude than the error created if both terms were excluded (Brunner, 1992). Therefore, the quasi-steady approximation is not used in channel routing.

7.5. Complete Hydraulic Models (Fully Dynamic Routing)

Description of Method

Complete hydraulic models solve the full Saint Venant equations simultaneously for unsteady flow along the length of a channel. They provide the most accurate solutions available for calculating an outflow hydrograph while considering the effects of channel storage and wave shape (Bedient and Huber, 1988). The models are categorized by their numerical solution schemes which include characteristic, finite difference, and finite element methods.

Characteristic methods were used for early numerical flood routing solutions based on the characteristic form of the governing equations. The two partial differential equations are replaced with four ordinary differential equations and solved along the characteristic curves (Henderson, 1966). The four equations are commonly solved using explicit or implicit finite difference techniques (Amein, 1966; Liggett and Woolhiser, 1967; Baltzer and Lai, 1968; Ellis, 1970; Strelkoff, 1970). Bedient and Huber (1988) state that characteristic methods incorporate cumbersome interpolations with no added accuracy compared to the finite difference techniques.

The finite difference method describes each point on a finite grid by the two partial differential equations and solves them using either an explicit or implicit numerical solution technique. Explicit methods solve the equations point by point in space and time along one time line until all the unknowns are evaluated then advances to the next time line (Fread, 1985). Much research has been performed on this topic (Garrison et al., 1969; Liggett and Woolhiser, 1967). Implicit methods simultaneously solve the set of equations for all points along a time line and then proceed to the next time line (Liggett and Cunge, 1975). Again, this topic has been well researched by Amein and Chu (1975), Amein and Fang (1970), and Fread (1973a and 1973b), among others. The implicit method has fewer stability problems and can use larger time steps than the explicit method.

Finite element methods can be used to solve the Saint Venant equations (Cooley and Mom, 1976). The method is commonly applied to two-dimensional models.

Assumptions

The assumptions given above for all hydraulic models (one-dimensional flow, fixed channel, constant density, and resistance described by empirical coefficients) apply to dynamic routing. It is also assumed that the cross sections used in the model fully describe the river's geometry, storage, and flow resistance.

Limitations

The major drawback to fully dynamic routing models is that they are time-consuming and data-intensive, and the numerical solutions often fail to converge when rapid changes (in time or space) are being modeled. This can be addressed by adjusting the time and distance steps used in the model; sometimes, however, memory or computational time limits the number of time and distance steps that may be used. Additionally, fully dynamic one-dimensional routing models do not describe situations (such as lakes and major confluences) where lateral velocities and forces are important.

Data Requirements

The accuracy of the model depends on the detail and accuracy of the river geometry that is input to the model (as well as the choice of appropriate time and distance steps). Input data for each cross section must describe channel slope and geometry; overbank storage; natural and man-made constrictions (such as bridges); channel and overbank roughness coefficients, and lateral inflows or outflows. In addition each model needs upstream and downstream “boundary conditions” - usually a flow hydrograph at the upstream end and some form of stage-discharge relationship at the downstream end.

Development of Equations

Dynamic routing models use finite-difference versions of the full St. Venant equations. This produces a set of simultaneous equations for each distance and time step, which are solved by a variety of techniques in different models.

Use and Estimation of Parameters

The hydraulic input data needed in a dynamic routing model is usually determined from a topographic map or surveyed river/valley cross sections. Any other special hydraulic conditions (upstream, downstream, or internal boundary conditions such as dams or waterfalls) must also be identified and described as a rating curve or fixed stage or flow hydrograph.

The selection of computational time and distance steps is critical to the accurate solution of the equations and to the numerical stability of the solution technique. A commonly cited guideline is that the wave speed c should be greater than the ratio of model distance step to time step.

8. Discussion of Routing methods

Routing method accuracy and application depends on basic assumptions inherent in the method. Researchers have performed studies trying to determine which routing method is best suited to a certain routing situation. Strelkoff (1980) compared the kinematic model, diffusion model, and modified Puls method with the complete Saint Venant equations as a basis for comparison. Floodplains, simple channels, and downstream boundary conditions were considered in the study. He found that the diffusion model produced results most comparable to the Saint Venant equations. The other methods produce acceptable results under certain conditions, as described in the preceding sections.

Using a one-dimensional unsteady flow model, UNET, as a basis, Brunner (1991) compared results from the Muskingum, Modified Puls, Muskingum-Cunge, and kinematic wave methods to those from UNET. Hypothetical river reaches (prismatic channels) and hydrograph data were used. Brunner determined that the hydrologic and hydraulic models tested are generally applicable for

channel slopes greater than 10 feet per mile (ft/mile). However, the hydrologic models typically failed when modeling slopes of 2 ft/mile or less. According to Brunner, the Muskingum-Cunge method was the most reliable of the hydrologic methods since it is partially derived from the momentum equation. He concluded that for slopes less than 2 ft/mile, only the full unsteady flow model should be used.

Long (1992) compared the Muskingum, Modified Puls, Muskingum-Cunge, modified attenuated kinematic wave, and variable travel time methods with the complete Saint Venant equations. She used two prismatic channels incorporating overbank storage and no overbank storage in addition to three different hydrograph types. Long found the Muskingum method produced the most comparable results to those of the Saint Venant equations with respect to peak flow, timing, and volume.